Mon. Not. R. Astron. Soc. **000**, 1–?? () Printed 19 January 2013 (MN LaTeX style file v2.2)

A Consistent Test of the Distance-Duality Relation with Galaxy Clusters and Type Ia Supernave

Nan Liang[1,2] ⋆, Shuo Cao[2], Kai Liao[2], and Zong-Hong Zhu[2] †
[1] *Center for High Energy Physics, Peking University, Beijing 100871, China*
[2] *Department of Astronomy, Beijing Normal University, 100875, Beijing, China*

22 Nov 2011

ABSTRACT

In this *letter*, we propose a new consistent method to revisit tests of the distance-duality (DD) relation which related angular diameter distances (D_A) to the luminosity distances (D_L) in a cosmology-independent way. In order to avoid any bias brought by redshift incoincidence between galaxy clusters and Type Ia Supernave (SNe Ia), as well as to ensure the integrity of the ADDs from galaxy clusters samples, we obtain the luminosity distance of one certain SN Ia point at the same redshift of the corresponding galaxy cluster by interpolating from the nearby SNe Ia. With the observational data at the same redshifts of the ADDs from the galaxy cluster sample for the spherical model and the corrected luminosity distances interpolated from the Union2 set, we find that $\eta \equiv D_L(1+z)^{-2}/D_A = 1$ is satisfied within 2σ confidence level for various parameterizations of $\eta(z)$, which are more stringent than previous testing results with redshift bias. We conclude that the DD relation is compatible with current observations of galaxy clusters for the spherical model and the Union2 set of SNe Ia.

Key words: (cosmology:) distance scale — galaxies: clusters: general — supernovae: general

1 INTRODUCTION

The Distance-duality (DD) relation, also known as the Etherington's reciprocity relation (Etherington 1933), is related the angular diameter distance (ADD, D_A) to the luminosity distance (D_L) by means of a single parameter,

$$\eta \equiv \frac{D_L}{D_A}(1+z)^{-2} = 1. \tag{1}$$

This equation is completely valid for all cosmological models based on Riemannian geometry (Ellis 2007). Therefore, the DD relation plays an essential role in observational astrophysics and modern cosmology (Csaki et al. 2002), such as galaxy clusters observations (Lima et al. 2003; Cunha et al. 2007), the anisotropies of cosmic microwave background (CMB) (Komatsu et al. 2011), as well as gravitational lensing studies (Schneider et al. 1999; Fu et al. 2008).

In principle, if both D_A and D_L of cosmological sources at the common redshifts are known, the DD relation ($\eta = 1$) could be directly tested by means of astronomical observations. From Sunyaev-Zeldovich effect (SZE, Sunyaev & Zel'dovich 1972) and X-ray surface brightness of galaxy clusters, the observational ADDs of galaxy clusters can be obtained (Silk & White 1978). By using an isothermal spherical model for which the hydrostatic equilibrium model and spherical symmetry assumed, Reese et al. (2002) selected 18 galaxy cluster sample and Mason et al. (2001) obtained seven clusters from the X-ray-limited flux sample. The measurements of the two samples above have been corrected by using an isothermal elliptical model to get 25 ADDs of galaxy clusters (De Filippis et al. 2005). Recently, Bonamete et al. (2006) obtained 38 ADD galaxy clusters sample by assuming the spherical model.

Uzan et al. (2004) considered ADDs of 18 galaxy cluster sample (Reese et al. 2002) to test the DD relation by assuming the ΛCDM model via the technique, $D_A^{\rm cluster}(z) = D_A^{\Lambda{\rm CDM}}(z)\eta^2(z)$. They showed that no violation of the DD relation is only marginally consistent. De Bernardis et al. (2006) considered ADDs of 38 galaxy cluster for spherical model (Bonamete et al. 2006) to test the DD relation by assuming the ΛCDM model. Some other works in context of the ΛCDM model for astrophysical research of the DD relation can be found in Bassett & Kunz (2004a), Bassett & Kunz (2004b), More et al. (2009), Avgoustidis et al. (2010), Holanda, Lima & Ribeiro (2011a), and Cao & Zhu (2011).

In order to test the DD relation in a model-independent way, one should use measurements of D_L such as Type Ia Supernave (SNe Ia) directly. By binning ADDs from FRIIb

⋆ liangn@bnu.edu.cn
† zhuzh@bnu.edu.cn

radio galaxies and ultra compact radio sources and D_L of SN Ia data, **?** found that the brightening excess of SNe Ia at $z > 0.5$ could cause a moderate violation at 2σ confidence level (CL). De Bernardis et al. (2006) binned ADD data of galaxy clusters (Bonamete et al. 2006) and the SNe Ia data to find that the validity of $\eta = 1$ is consistent at 1σ CL. However, it is argued that the above tests may have been influenced by the particular choice of redshift bin (Holanda, Lima & Ribeiro 2010).

Recently, Holanda, Lima & Ribeiro (2010) tested the DD relation with two ADD samples (Bonamete et al. 2006; De Filippis et al. 2005) and the Constitution set of SNe Ia data (Hicken et al. 2009). For the biggest redshift difference between clusters and SNe Ia is $\Delta z = |z_{\rm cluters} - z_{\rm SNe}| \simeq 0.01$ for three clusters, a selection criteria ($\Delta z \leqslant 0.005$) for a given pair of data set are used to avoid the corresponding bias of redshift differences. With the incomplete spherical model sample (Bonamete et al. 2006) in which three ADD data have been discarded, they found a strong violation ($> 3\sigma$) of the DD relation by using two parameterizations of η parameter [$\eta(z) = 1 + \eta_1 z$, and $\eta(z) = 1 + \eta_a z/(1+z)$].

More recently, Li, Wu & Yu (2011) used the same selection criteria for given pairs of observational data to remove more data points of the galaxy clusters corresponding to the Constitution set and found that the DD relation could be marginally accommodated at 3σ CL for the spherical model if the effect of the errors of SNe Ia considered. Additionally, they also examined the DD relation for two more general parameterizations [$\eta(z) = \eta_0 + \eta_1 z$, and $\eta(z) = \eta_0 + \eta_1 z/(1+z)$] to show that $\eta(z) = 1$ is compatible with the spherical model sample and the Union2 set (Amanullah et al. 2010) at 2σ CL. Some recent works for testing the DD relation can be found in, e. g., Nair et al. (2011); Cao & Liang (2011); Meng et al. (2011); Holanda, Lima & Ribeiro (2011b); Fu et al. (2011).

It is obvious that testing results of the DD relation may be influenced by the particular choice of the selection criteria for a given pair of data set. The difference of redshifts between pairs of galaxy clusters and SNe Ia may cause obvious deviation in testing the DD relation. In principle, the only strict criterion to form a given pair is that galaxy clusters and SNe Ia locate at exactly the same redshift. At other hand, the more stringent selection criteria are used, the more data points should be removed. In order to avoid any bias of redshift differences between SNe Ia and galaxy clusters and ensure the integrity of observational data pairs, we can use the nearby SNe Ia points to obtain the luminosity distance of SN Ia point at the same redshift of the corresponding galaxy cluster; this situation is similar with the cosmology-independent calibration of GRB relations directly from SNe Ia (Liang et al. 2008; Liang, Wu & Zhang 2010; Liang, Xu & Zhu 2011).

In this *letter*, we test the DD relation with the Union2 set in which a sub-sample of SNe Ia are corrected to the same redshifts of the corresponding galaxy clusters sample by interpolating from the nearby SNe Ia points with the biggest difference of redshifts $\Delta z_{\rm max} = 0.005$ for a given pair of data set. We focus on the 38 ADDs from galaxy cluster sample under an assumption of spherical model (Bonamete et al. 2006). As we well see, there exists no conceivable evidence for variations in the DD relation when current observations are confronted, since $\eta(z) = 1$ is significant satisfied at 2σ confidence level for various parameterizations of $\eta(z)$.

2 DATA ANALYSIS

In this work, we test the DD relation with the 38 ADD sample from galaxy clusters for the spherical model (Bonamete et al. 2006) and the Union2 set which consists of 557 SNe Ia (Amanullah et al. 2010). It is easy to find that differences of redshifts between the 38 galaxy clusters to the Union2 set are more centered around $\Delta z = 0$ and the biggest value at $\Delta z = 0.005$ for a given pair of data set; this situation can provide the accuracy in the interpolating procedure. Therefore, we can obtain the luminosity distance of SN Ia at the same redshift of the corresponding galaxy cluster by interpolating from the nearby SNe Ia points with the biggest difference of redshifts $\Delta z = 0.005$ for a given pair of data set. Obviously, our method can successfully avoid the systematic errors brought by redshift incoincidence of the observational data pairs and ensure the integrity of observational data pairs.

If the DD relation is considered be in a redshift-dependent form, the observation technique gives $D_A^{\rm cluster}(z) = D_A(z)\eta^2(z)$ (Cavaliere & Fusco-Fermiano 1978), therefore, $D_A(z)$ must be replaced with $D_A^{\rm cluster}(z)\eta^{-2}$ when tested the DD relation consistently with the SZE+X-ray observations from galaxy clusters (Holanda, Lima & Ribeiro 2010). The observed $\eta_{obs}(z)$ can be determined by

$$\eta_{obs}(z) = (1+z)^2 \frac{D_A^{\rm cluster}(z)}{D_L^{\rm corrected}(z)}. \tag{2}$$

where $D_A^{\rm cluster}$ is ADD from galaxy cluster at redshift z inside the samples, and $D_L^{\rm corrected}$ is the corrected luminosity distance interpolated from the nearby SNe Ia points $D_L^{\rm SNe}$. In the interpolating procedure, we weighted the SNe data at the same redshifts each other

$$\bar{\mu}(z) = \frac{\sum\left(\mu_i/\sigma^2_{\mu_i}\right)}{\sum 1/\sigma^2_{\mu_i}}, \tag{3}$$

where $\bar{\mu}(z)$ stands for the weighted mean distance modulus at the same redshift z with its uncertainty $\sigma_{\bar{\mu}} = (\sum 1/\sigma^2_{\mu_i})^{-1/2}$. We note that the data points of the Union2 set are given in terms of the distance modulus, which could reduce to the luminosity distance by $D_L(z) = 10^{\mu(z)/5-5}$. Accordingly, the uncertainty of the luminosity distance could be expressed as $\sigma_{D_L(z)} = (\ln 10/5)D_L(z)\sigma_{\mu(z)}$.

The DD relation can be tested with the combined observational data by the minimum χ^2 method. The total χ^2 can be given by

$$\chi^2(\mathbf{p}) = \sum_z \frac{[\eta(z;\mathbf{p}) - \eta_{\rm obs}(z)]^2}{\sigma^2_{\eta_{\rm obs}}}, \tag{4}$$

where $\eta(\mathbf{p})$ represents the theoretical value with the parameter set $\mathbf{p}$, and $\eta_{\rm obs}$ associated with the observational technique with its error $\sigma_{\eta_{\rm obs}}$, which comes from the uncertainties of ADDs (σ_{D_A}) and the corrected luminosity distances ($\sigma_{D_L^{\rm corrected}}$)

$$\sigma^2_{\eta_{\rm obs}} = \eta^2_{\rm obs}[(\sigma_{D_A^{\rm cluster}}/D_A^{\rm cluster})^2 + (\sigma_{D_L^{\rm corrected}}/D_L^{\rm corrected})^2], \tag{5}$$

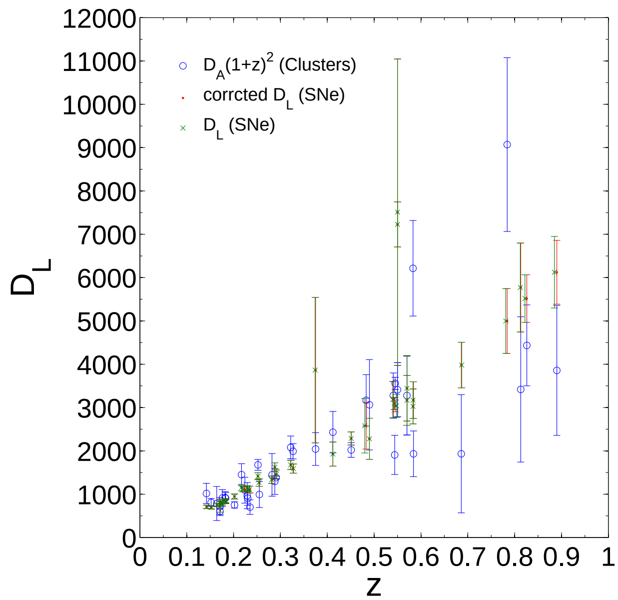

Figure 1. Galaxy clusters and the corresponding SNe Ia data with the associated error bars. The blue open circles and red filled circles stand $(1+z^2)D_A$ from the galaxy clusters for the spherical model and the corresponding corrected D_L from the nearby Union2 sub-sample, green '×' stand D_L directly from the Union2 sub-sample.

Following Holanda, Lima & Ribeiro (2010); Li, Wu & Yu (2011), we combine the statical and systematic uncertainties of galaxy clusters in quadrature (D'Agostini. 2004). The asymmetry uncertainties of galaxy clusters can be treated by an statistical approach (D'Agostini. 2004), and the topic systematic uncertainties of galaxy clusters are around +12.4% and −12% (Bonamete et al. 2006). In Figure 1, we plot D_A data from the galaxy cluster and the corresponding corrected D_L data from Union2 sub-sample at the same redshifts of galaxy clusters.

3 RESULTS

In Figure 2, we show testing results of the DD relation with ADDs and the Union2 set by considering one-parameter parameterizations [$\eta(z) = 1 + \eta_1 z$ (Figure 2 *Left*) and $\eta(z) = 1 + \eta_a z/(1+z)$ (Figure 2 *Right*)]. For comparison, the case with the corrected luminosity distance ($D_L^{\rm corrected}$) interpolated from the nearby SNe Ia and the case with the Union2 set directly ($D_L^{\rm SNe}$) are given simultaneously. For the case with $D_L^{\rm corrected}$, the best-fit values are $\eta_1 = -0.204 \pm 0.215$ at 2σ CL with $\chi^2_{\rm min} = 29.00$, and $\eta_a = -0.302 \pm 0.328$ at 2σ CL with $\chi^2_{\rm min} = 29.24$. For the case with the Union2 set directly, the best-fit values are $\eta_1 = -0.224 \pm 0.215$ (2σ), and $\eta_a = -0.334 \pm 0.215$ (2σ). In order to compare with previous results from the incomplete ADD sample and the Constitution set, we also show testing results with complete ADD sample and the Constitution set in Figure 3. For the case with $D_L^{\rm corrected}$, the best-fit values are $\eta_1 = -0.431 \pm 0.303$ at 3σ CL with $\chi^2_{\rm min} = 33.10$, and $\eta_1 = -0.664 \pm 0.457$ at 3σ CL with $\chi^2_{\rm min} = 32.34$. For the case with the Constitution set directly, the best-fit values are $\eta_1 = -0.517 \pm 0.286$ (3σ), and $\eta_a = -0.793 \pm 0.436$ (3σ). Fitting results with the 38 ADDs of galaxy clusters and the corrected luminosity distances of the Union2 set and the Constitution set are summarized in Table 1.

Our results of the case with the Union2 set directly are consistent with those obtained by Li, Wu & Yu (2011), where $\eta_1 = -0.22 \pm 0.21$ and $\eta_a = -0.33 \pm 0.33(2\sigma)$. From comparing to results of the case with the corrected luminosity distance and the case with SN Ia set (the Union2 set and the Constitution set) directly, we can see a shift between the best fit values and the likelihood contours towards the standard DD relation ($\eta = 1$) with lower $\chi^2_{\rm min}$ for using of the interpolating method to obtain $D_L^{\rm corrected}$. This situation shows that the using of the interpolating method tend to avoid the corresponding bias of redshift differences and make results be more compatible with the DD relation. Compared to results of the case with the the Union2 set and the case with Constitution set, it is shown that that the DD relation of the one-parameter parameterizations with the Union2 set for the interpolating method is well satisfied within 2σ CL; while the DD relation is inconsistent with the Constitution set for both cases at 3σ CL. Compared to previous results with the incomplete ADD sample and the Constitution set, our analyses with the complete spherical model sample (38 ADDs) and the Constitution set directly are consistent with previous results obtained by Holanda, Lima & Ribeiro (2010) with the incomplete spherical model sample (35 ADDs, three points removed by selection criteria) and the Constitution set, where $\eta_1 = -0.42 \pm 0.34$, and $\eta_a = -0.66 \pm 0.50$ at 3σ CL; and inconsistent with those obtained by Li, Wu & Yu (2011) with the incomplete spherical model sample (26 ADDs, 12 points removed by selection criteria) and the Constitution set, where $\eta_1 = -0.30 \pm 0.34$ and $\eta_a = -0.46 \pm 0.51(3\sigma)$. This situation shows that the choice of selection criteria to remove ADD points with large bias of redshift differences may paly an important role in testing of the DD relation.

Parameterization (SN Ia*)	η_1/η_a	$\chi^2_{\rm min}$	$\chi^2_{\rm dof}$
$1+\eta_1 z$ (Union2*)	$\eta_1 = -0.204 \pm 0.215(2\sigma)$	29.00	0.78
$1+\eta_1 z$ (Union2)	$\eta_1 = -0.228 \pm 0.211(2\sigma)$	29.32	0.79
$1+\eta_a \frac{z}{1+z}$ (Union2*)	$\eta_a = -0.302 \pm 0.329(2\sigma)$	29.24	0.79
$1+\eta_a \frac{z}{1+z}$ (Union2)	$\eta_a = -0.334 \pm 0.333(2\sigma)$	29.76	0.80
$1+\eta_1 z$ (Constitution*)	$\eta_1 = -0.431 \pm 0.303(3\sigma)$	33.10	0.89
$1+\eta_1 z$ (Constitution)	$\eta_1 = -0.517 \pm 0.286(3\sigma)$	40.97	1.10
$1+\eta_a \frac{z}{1+z}$ (Constitution*)	$\eta_a = -0.664 \pm 0.457(3\sigma)$	32.33	0.87
$1+\eta_a \frac{z}{1+z}$ (Constitution)	$\eta_a = -0.793 \pm 0.436(3\sigma)$	40.46	1.09

Table 1. Fitting results with the 38 ADDs of galaxy clusters and the Union2 set and Constitution set, and $\chi^2_{\rm min}$ (the minimun χ^2), $\chi^2_{\rm dof}$ ($\chi^2_{\rm min}$/dof), for $\eta(z) = 1 + \eta_a z$ and $\eta(z) = 1 + \eta_a \frac{z}{1+z}$, respectively. The asterisk represents the case with the corrected luminosity distance interpolated from the nearby SNe Ia.

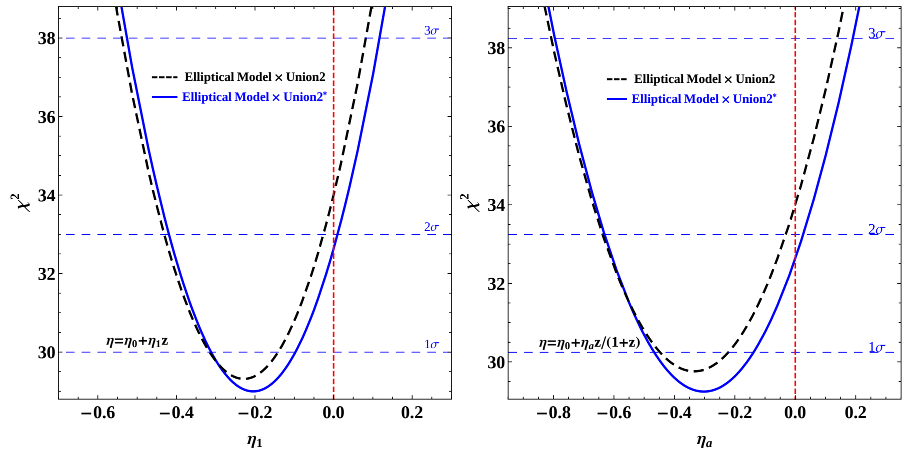

Figure 2. Likelihood contours with the 38 ADDs of galaxy clusters and the corrected luminosity distances of the Union2 set in the $\eta_1 - \Delta\chi^2$ plane (*Left*: for $\eta(z) = 1 + \eta_a z$), and in the $\eta_a - \Delta\chi^2$ plane (*Right*: for $\eta(z) = 1 + \eta_a \frac{z}{1+z}$). The blue real lines represent the case with the corrected luminosity distance interpolated from the nearby SNe Ia (Union2*), the black dashed lines represent the case with the SNe Ia set (Union2) directly, and the red vertical lines represent $\eta(z) = 1$.

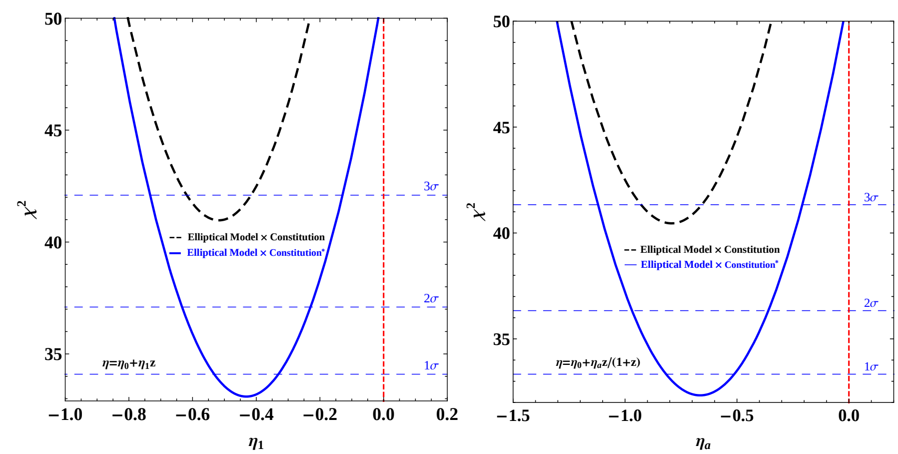

Figure 3. Likelihood contours with the 38 ADDs of galaxy clusters and the corrected luminosity distances of the Constitution set in the $\eta_1 - \Delta\chi^2$ plane (*Left*: for $\eta(z) = 1 + \eta_a z$), and in the $\eta_a - \Delta\chi^2$ plane (*Right*: for $\eta(z) = 1 + \eta_a \frac{z}{1+z}$). The blue real lines represent the case with the corrected luminosity distance interpolated from the nearby SNe Ia (Constitution*), the black dashed lines represent the case with the SNe Ia set (Constitution) directly, and the red vertical lines represent $\eta(z) = 1$.

4 CONCLUSIONS

In this *letter*, we preform a new consistent test for the distance-duality relation $[\eta(z) \equiv D_L(1+z)^{-2}/D_A{=}1]$ in a cosmology-independent way. It is obvious that the redshift differences of observational samples may cause deviation of the DD relation. Testing results from given pairs of data set with the corresponding galaxy clusters and SNe Ia at nearby redshift may be influenced by the particular choice of the selection criteria; the more stringent selection criteria are used, the more data points should be removed. In order to avoid any bias of difference of redshift and ensure the integrity of the ADD samples, we correct the luminosity distance of a SN Ia to the same redshift of the corresponding galaxy cluster directly from the nearby SN Ia points. With the 38 ADD sample from galaxy clusters under an assumption of spherical model and the corrected luminosity distances of the Union2 set, fitting results of the DD relation are $\eta_1 = -0.204{\pm}0.215$ at 2σ CL for parameterization $\eta(z) = 1 + \eta_a z$, and $\eta_a = -0.304{\pm}0.215$ at 2σ CL for parameterization $\eta(z) = 1 + \eta_a \frac{z}{1+z}$, respectively.

Our results show that there exists no conceivable evidence for variations in the duality distance relation when the current SNe Ia and the complete sample of galaxy clusters data are confronted, since various parameterizations of $\eta(z)$ are significant satisfied at 2σ CL by using the interpolating method, which are more stringent than those obtained in Li, Wu & Yu (2011), where the DD relation is only marginally accommodated at 3σ CL for the spherical model sample. Compared to previous testing results with redshift bias and incomplete sample, our results are inconsistent with those obtained in Holanda, Lima & Ribeiro (2010), where the spherical model sample give a clear violation of the DD relation ($> 3\sigma$). We conclude that the DD relation is significantly compatible with current observations of galaxy clusters for the spherical model and the Union2 set of SNe Ia.

ACKNOWLEDGMENTS

We thank the anonymous referee for constructive suggestions and valuable comments. N. L. thanks Puxun Wu, Zhixiang Li and Hao Wang for discussions and help for calculations. Z-H Z. acknowledges the National Natural Science Foundation of China under the Distinguished Young Scholar Grant 10825313 and Grant 11073005, the Ministry of Science and Technology national basic science Program (Project 973) under Grant No.2012CB821804 , and "the Fundamental Research Funds for the Central Universities".

REFERENCES

Amanullah, R., et al., 2010, ApJ, 716, 712
Avgoustidis, A. et al. 2010, JCAP, 10, 024
Bassett, B. A., & Kunz, M. 2004a, ApJ, 607, 661
Bassett, B. A., & Kunz, M. 2004b, PRD, 69, 101305
Bonamente, M., et al. 2006, ApJ, 647, 25
Cao, S., & Zhu, Z.-H. 2011, [arXiv:1102.2750]
Cao, S., & Liang, N. 2011, RAA, 11, 1199
Cavaliere, A. & Fusco-Fermiano, R. 1978, A&A, 667, 70
Csaki, C., Kaloper, N. & Terning, J. 2002, PRL, 88, 161302
Cunha, J. V., Marassi, L., & Lima, J. A. S. 2007, MNRAS, 379, L1
D'Agostini, G. 2004, arXiv:phisics/040308v2
De Bernardis, F., Giusarma, E., & Melchiorri, A. 2006, IJMPD, 15, 759
De Filippis et al. 2005, ApJ, 625, 108
Ellis, G. F. R. 2007, Gen. Rel. Grav., 39, 1047
Etherington, I. M. H. 1933, Phil. Mag. 15, 761; reprinted in 2007, Gen. Rel. Grav. 39, 1055
Fu, L., et al. 2008, A&A, 479, 9
Fu, X. Y., Wu, P. X., Yu, H. W., & Li, Z. 2011, RAA,11, 8
Hicken, M., et al. 2009, ApJ, 700, 1097
Holanda, R. F. L., Lima, J. A. S., & Ribeiro, M. B., 2010, ApJ, 722, L233
Holanda, R. F. L., Lima, J. A. S., & Ribeiro, M. B., 2011, A&A, 528, 14
Holanda, R. F. L., Lima, J. A. S., & Ribeiro, M. B., arXiv:1104.3753
Komatsu, E., et al., 2011, ApJS, 192, 18 [arXiv:1001.4538]
Lima, J. A. S., Cunha, J. V., & Alcaniz, J. S. 2003, PRD, 68, 023510
Li, Z. X., Wu, P. X., & Yu, H. W. 2011, ApJ, 729, L14
Liang, N. et al. 2008, ApJ, 685, 354
Liang, N., Wu, P., & Zhang, S. N. 2010, PRD, 81, 083518
Liang, N., Xu, L., & Zhu, Z.-H. 2011, A&A, 527, A11
Mason, B. S., et al. 2001, ApJ, 555, L11
Meng, X.-L., Zhang, T.-J., & Zhan, H. 2011, arXiv:1104.2833
More, S., Bovy, J., & Hogg, D. W. 2009, ApJ, 696, 1727
Nair, R., Jhingan, S., & Jain, D. 2011 arXiv:1102.1065
Reese, E. D., et al. 2002, ApJ, 581, 53
Schneider, P., Ehlers, J., & Falco, E. E. 1999, Gravitational Lenses (Springer-Verlag, Berlin)
Silk, J., & White, D. M., 1978, ApJL, 226, L103
Sunyaev, R. A., Zel'dovich, Ya. B. 1972, Comm. Astrophys. Space Phys., 4, 173
Uzan, J. P., Aghanim, N., & Mellier, Y. 2004, Phys. Rev. D, 70, 083533